Intersecting Faults Simulation for Three-Dimensional Reservoir-Geomechanical Models

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Abstract

Faults are geological entities with thicknesses (of the order cm) several orders of magnitude smaller than the grid blocks (of the order 10 m) typically used to discretize reservoir and/or over-under-burden geological formations. Introducing faults in a complex reservoir and/or geomechanical mesh therefore poses significant meshing difficulties. In this paper, we introduce faults in the mesh without meshing them explicitly, by using the extended finite element method (X-FEM) in which the nodes whose basis function support intersects the fault are enriched within the framework of partition of unity. For the geomechanics, the fault is treated as an internal displacement discontinuity that allows slipping to occur using a Mohr-Coulomb type criterion. For the reservoir, the fault is either an internal fluid flow conduit that allows fluid flow in the fault as well as to enter/leave the fault or is a barrier to flow (sealing fault). Equal-order displacement and pressure approximations are used. Two- and three-dimensional benchmark computations are presented to verify the accuracy of the approach, and simulations are presented that reveal the influence of the rate of loading on the activation of faults.

INTRODUCTION

Faults are geological entities with thicknesses several orders of magnitude smaller than the grid blocks typically used to discretize reservoir and/or over-under-burden geological formations. Coates and Schoenberg (1995) used finite-difference to model faults with a displacement discontinuity across it. Since this initial work, finite elements have been adopted for faults modeling. However, introducing faults in a complex reservoir and/or geomechanical finite element mesh presents significant difficulties due to the need to generate very refined meshes in the vicinity of the faults. Several researchers have recently focused on fault modeling in geomaterials (see, e.g. Cappa and Rutqvist 2011; Rinaldi et al. 2014), but most of the studies are limited to two dimensions and only approximately account for the coupling between fluid flow and solid deformation that occurs in fluid-saturated porous media (so-called poro-mechanical effects). Furthermore, these implementations are restricted to sealing faults, and do not fully address the challenge of inserting a fault within a mesh. A recent study (Jha and Juanes 2014) addresses this challenge in both 2D and 3D by modeling faults as surfaces of discontinuity using interface elements (fault must conform to element boundaries) and Lagrange multipliers, but it is also restricted to sealing faults. Early theoretical work on issues related to embedding strong discontinuities in saturated porous materials can be found in Armero and Callari (1999). In the present paper, we use the extended finite element method (X-FEM) to introduce faults without the need to mesh them explicitly. Numerical results using the X-FEM have been presented in 2D
for fractured porous media (de Borst et al. 2006; Fumagalli and Scotti 2014; Lamb et al. 2013; Réthoré et al. 2007; Talebian et al. 2013), and for 3D hydraulic fracture simulations (Secchi and Schrefler 2012; Gupta and Duarte 2014). The implementation of the X-FEM in 3D (Sukumar et al. 2000) is significantly more complex than in 2D (Moës et al. 1999).

The proposed approach is a marked departure from the FEM for the modeling of faults. Significant meshing difficulties arise when faults (multiple) are modeled using the FEM, where the mesh has to conform to the geometry of the fault(s) and hence a structured mesh cannot be used. Unstructured mesh generation (triangles in 2D and tetrahedra in 3D) is needed for the FEM. On using the X-FEM, a structured mesh suffices and the faults can arbitrarily cut the elements in the mesh. This is illustrated in Figure 1 where a fault with thickness 0.1 m is inserted into a zone 20mx20m. The FEM mesh shown in Figure 1a uses an unstructured mesh with 19,724 nodes and 39,366 triangular elements (30,140 for the domain and 9,226 for the fault). Figure 1b shows a zoom of the mesh next to the fault. The X-FEM mesh is shown in Figure 1c and uses only 81 nodes and 64 quad elements. The X-FEM enriched nodes are shown as open circles in Figure 1c. Clearly, the savings achieved by the X-FEM procedure are VERY substantial. Further, since the fault is not meshed, inserting many (tens or even hundreds) of faults is relatively straightforward since the faults are never meshed and hence they can be placed within the reservoir mesh by simply identifying their plane(s). In this presentation we extend the work presented in Prévost and Sukumar (2016) and consider nonplanar and intersecting faults.

Figure 1. (a) FEM meshing of fault, (b) zooming on FEM fault, (c) X-FEM mesh: Enriched nodes (shown as open circles) whose basis function support is cut by the fault \( \Gamma_c \).

**FAULTS MODELING WITH THE X-FEM**

The discretized finite element equations are obtained by using finite element basis functions that are augmented by X-FEM functions to represent the fault \( \Gamma_c \). Let \( N^i(\mathbf{x}) \) denote the standard finite element shape functions for a single finite element, where \( i \in \mathbb{N} \), the set consisting of nodes that belong to the element. Let \( \mathbb{N}_{cut} \) denote the set of nodes whose basis function support is cut by the fault \( \Gamma_c \) (see Figure 1c). Then the following approximations are used:
a discontinuous displacement field (stress equation) (Moës et al. 1999):
\[
\mathbf{u} = \sum_{i \in \mathbb{N}} N_i^i(x) \mathbf{u}_i + \sum_{i \in \mathbb{N}_{cut}} N_i^i(x) H_{\Gamma_e}(x) \hat{\mathbf{u}}_i,
\]
(1)
where \(H_{\Gamma_e}(x)\) is the discontinuous generalized Heaviside function;

- a continuous pressure field (with fluid as a flow conduit):
\[
p_f = \sum_{i \in \mathbb{N}} N_i^i(x) p_f^i + \sum_{i \in \mathbb{N}_{cut}} N_i^i(x) \psi(x) \hat{p}_f^i,
\]
(2)
where \(\psi(x)\) is the distance function with a discontinuous normal derivative across \(\Gamma_e\);

- and a discontinuous pressure field (with a sealing fault):
\[
p_f = \sum_{i \in \mathbb{N}} N_i^i(x) p_f^i + \sum_{i \in \mathbb{N}_{cut}} N_i^i(x) H_{\Gamma_e}(x) \hat{p}_f^i.
\]
(3)

For the stress equation each node \(i\) is assigned \(nsd\) (number of spatial dimensions) displacement degrees of freedom \(\mathbf{u}_i\). The set of nodes \(\mathbb{N}_{cut}\), whose basis function support is cut by the fault are assigned additional \(nsd\) degrees of freedom \(\hat{\mathbf{u}}_i\). Similarly for the pressure equation, each node \(i\) is assigned one pressure degree of freedom \(p_f^i\) and the nodes in \(\mathbb{N}_{cut}\) are assigned an additional pressure degree of freedom \(\hat{p}_f^i\). The resulting semi-discrete finite element equations are integrated in time by using a first order finite difference time-stepping integrator (typically, backward Euler) for both stress and pressure equations.

**TECHNICAL CHALLENGES**

The contribution to the residuals of the elements that are cut by the fault require element partitioning above, below and on \(\Gamma_e\) for spatial integration. In 2D, we use triangles for surface integration and line segments for line integration on the fault as implemented in Sukumar and Prevost (2003), although there are now procedures available in 2D to integrate without requiring partitioning as proposed by Chin et al. (2016). The 2D recursive partitioning of a cell in which 2 faults intersect is illustrated in Figure 2. In 3D, we subdivide the cells by using tetrahedral elements (Sukumar et al. 2000) for volume integration and triangles for surface integration on the fault. For intersecting faults, the recursive implementation in 3D is VERY challenging.

**Figure 2. 2D recursive partitioning for intersecting faults**
TABLE 1. MATERIAL PROPERTIES

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus $E^s$</td>
<td>$30 \text{ GPa}$</td>
</tr>
<tr>
<td>Poisson’s ratio $\nu^s$</td>
<td>$0.25$</td>
</tr>
<tr>
<td>Rock permeability $k_{rock}$</td>
<td>$k = 10^{-15} \text{ m}^2$ (sandstones)</td>
</tr>
<tr>
<td>Fluid viscosity $\mu_f$</td>
<td>$10^{-9} \text{ MPa s}$</td>
</tr>
<tr>
<td>Fluid bulk modulus $K_f$</td>
<td>$2 \text{ GPa}$</td>
</tr>
<tr>
<td>Porosity $\varphi$</td>
<td>$0.30$</td>
</tr>
<tr>
<td>Fault permeability $k_u$</td>
<td>$k_u = 10^{-3} k_{rock}$</td>
</tr>
<tr>
<td>Fault friction angle $\varphi_u$</td>
<td>$30^\circ$</td>
</tr>
<tr>
<td>Major fault angle $\alpha$</td>
<td>$58^\circ$</td>
</tr>
<tr>
<td>Coefficient of diffusion $c_f$</td>
<td>$\frac{k}{\mu_f} \frac{K^s + \frac{4}{3} G^s}{K^s + \frac{4}{3} G^s + b^2 M} = 20 \text{ m}^2 / \text{hr}$</td>
</tr>
</tbody>
</table>

NUMERICAL RESULTS

Numerical simulations in 2D and 3D are presented, which demonstrate the accuracy and versatility of the proposed X-FEM fault models.

Fluid injection in a 2D domain

The problem geometry, initial and boundary conditions are shown in Figure 3. The material properties are listed in Table 1. The major fault is inclined at an angle $58^\circ$, close to the potential failure angle of $60^\circ$ (the fault friction angle is assumed to be $30^\circ$). An injection over pressure of $5 \text{ MPa}$ is assumed. Full poro-mechanical effects are taken into account and the time scale is controlled by the fluid coefficient of diffusion $c_f = 20 \text{ m}^2 / \text{hr}$ (Coussy 2004).

Failure and slip occurs along the major fault after 750 hrs of injection as shown in Figure 4. Also, note that as a result of the inclined fault slip, the vertical fault opens up as shown in Figure 5.
Figure 3. Problem geometry, initial and boundary conditions

Figure 4. Fault slip after 750 hrs of fluid injection, (a) solid displacement vectors; (b) solid displacement vectors norm; (c) deformed mesh.
Figure 5. Normal (red) and tangential (blue) motions along the fault. Note that the vertical fault opens up as a result of the inclined fault slip

Fluid inflow in a 3D domain

The problem geometry and boundary conditions are shown in Figure 6. The domain is assumed to be 400x400x400 m and the thickness of the faults is 4 cm. Note that 2 intersecting faults are present in the domain. Poro-mechanical effects are ignored in this example, and the pressure equation is solved as an elliptic equation (steady-state). Resulting Darcy velocities are shown in Figure 7 for both a structured and an unstructured mesh.

\[ q_f = 10^{-6} \text{ m/s} \]

faults:
\[ k_n = 10^{-3} k_{rock} \quad k_t = 10^{-3} k_{rock} \]

thickness = 10^{-3} h^* = 0.04 \text{ m}

Figure 6. Fluid inflow in a 3D domain – boundary conditions
hexa mesh: 1,334 nodes 1,000 cells

tetra mesh: 3,223 nodes 16,261 cells

(a) (b)

Figure 7. X-FEM solution for the fluid inflow in a 3D domain.
Two intersecting faults are present (a) Structured and unstructured meshes
(b) Darcy velocities; maximum 7.5 e-10 m/s

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REFERENCES


