PUFEM — Revisited

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September 18, 1996

1 Problem Statement

Consider the following boundary-value problem:

$$\begin{aligned} u'' &= 2 \quad \text{in } (0,\, 10) \\ u(0) &= 0 \\ u(10) &= 0\,. \end{aligned} \tag{1}$$

The exact solution of the above is:

$$\begin{aligned} u^{\text{exact}} &= x^2 - 10x \\ \varepsilon^{\text{exact}} &= 2x - 10\,. \end{aligned} \tag{2}$$

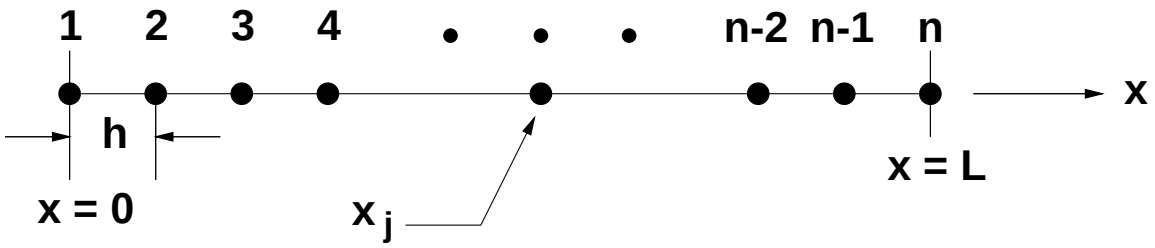

Figure 1: Nodal Discretization ($L = 10$, $n = 11$)

2 Numerical Results

The domain is discretized by 11 equi-spaced nodes (see Figure). In the computations, a quartic polynomial is used to form the partition of unity. Numerical integration is carried out using **ten-point** Gauss quadrature per cell. Results are presented in Tables 1 and 2 for non-shifted and shifted basis, respectively — identical results are obtained for the norms using the two approaches. In Figure 2 and 3, the PUFEM solution is compared to the exact solution for two particular cases.

Table 1: $\mathcal{L}_2$– and $\mathcal{H}^1$–norms [**non-shifted basis**]

Bases on node I			d_{max} **of node I**			$\mathcal{L}_2$–**Error**	$\mathcal{H}^1$–**Error**
1	$\mathbf{2-10}$	**11**	$\mathbf{1,2}$	$\mathbf{3-9}$	$\mathbf{10,11}$	**Norm**	**Norm**
$\{-10x, x^2\}$	$\{1, x, x^2\}$	$\{-10x, x^2\}$	0.90	0.90	0.90	2.6E-3	5.8E-3
$\{-10x, x^2\}$	$\{1, x, x^2\}$	$\{-10x, x^2\}$	0.99	0.99	0.99	1.3E-4	3.4E-4
$\{-10x, x^2\}$	$\{1, x, x^2\}$	$\{-10x, x^2\}$	1.00	1.00	1.00	1.6E-4	3.7E-4
$\{-10x, x^2\}$	$\{1, x, x^2\}$	$\{-10x, x^2\}$	1.00	1.10	1.00	3.2E-4	5.9E-4
$\{-10x, x^2\}$	$\{1, x, x^2\}$	$\{-10x, x^2\}$	1.00	1.50	1.00	7.3E-3	2.2E-2
$\{-10x, x^2\}$	$\{1, x, x^2\}$	$\{-10x, x^2\}$	1.00	1.99	1.00	1.8E-4	9.8E-4
$\{-10x, x^2\}$	$\{1, x\}$	$\{-10x, x^2\}$	1.00	1.00	1.00	2.6E-1	1.6E+0

Table 2: $\mathcal{L}_2$– and $\mathcal{H}^1$–norms [**shifted basis**]

Bases on node I			d_{max} **of node I**			$\mathcal{L}_2$–**Error**	$\mathcal{H}^1$–**Error**
1	$\mathbf{2-10}$	**11**	$\mathbf{1,2}$	$\mathbf{3-9}$	$\mathbf{10,11}$	**Norm**	**Norm**
$\{-10x, x^2\}$	$\{1, x, x^2\}$	$\{-10x, x^2\}$	0.90	0.90	0.90	2.6E-3	5.8E-3
$\{-10x, x^2\}$	$\{1, x, x^2\}$	$\{-10x, x^2\}$	0.99	0.99	0.99	1.3E-4	3.4E-4
$\{-10x, x^2\}$	$\{1, x, x^2\}$	$\{-10x, x^2\}$	1.00	1.00	1.00	1.6E-4	3.7E-4
$\{-10x, x^2\}$	$\{1, x, x^2\}$	$\{-10x, x^2\}$	1.00	1.10	1.00	3.2E-4	5.9E-4
$\{-10x, x^2\}$	$\{1, x, x^2\}$	$\{-10x, x^2\}$	1.00	1.50	1.00	7.3E-3	2.2E-2
$\{-10x, x^2\}$	$\{1, x, x^2\}$	$\{-10x, x^2\}$	1.00	1.99	1.00	1.8E-4	9.8E-4
$\{-10x, x^2\}$	$\{1, x\}$	$\{-10x, x^2\}$	1.00	1.00	1.00	2.6E-1	1.6E+0

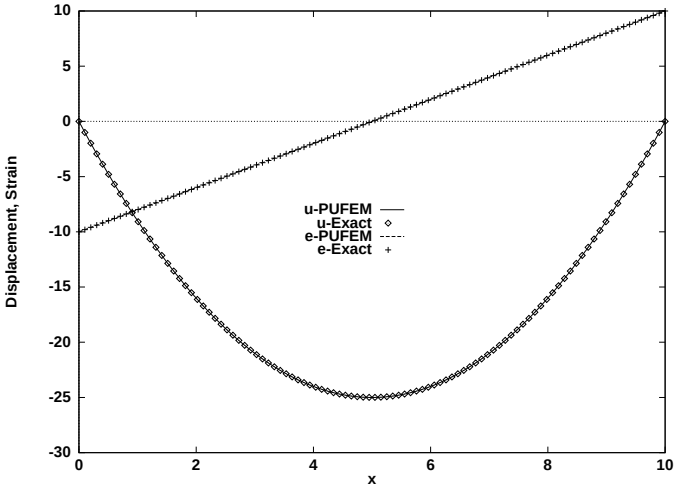

Figure 2: Results for $d_{max} = 0.9$ with quadratic basis on the interior nodes (non-shifted basis)

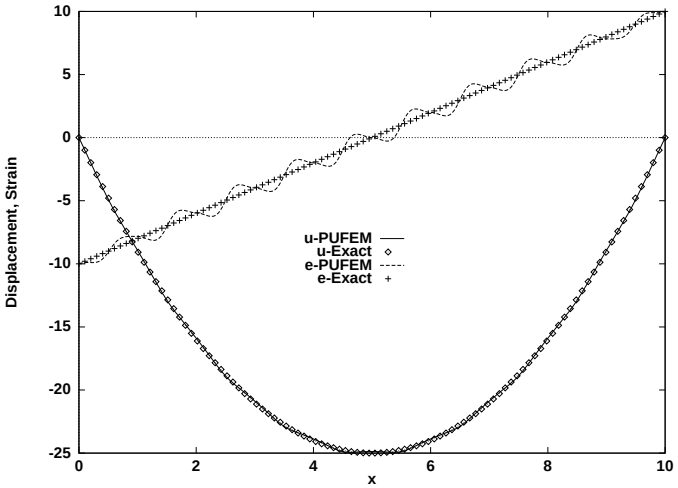

Figure 3: Results for $d_{max} = 1.0$ with linear basis on the interior nodes (non-shifted basis)