Patch Test in 1D: Coupled FE-EFG Method

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1 Patch Test: Problem Statement and Solution

Consider the following homogeneous one-dimensional Dirichlet boundary-value problem (strong form):

$$\begin{aligned} \frac{d}{dx}\left(E\frac{du}{dx}\right) &= 0 \quad \text{in } \Omega\,, \\ u(0) &= 0\,, \\ u(L) &= L\,, \end{aligned} \tag{1}$$

where $\Omega = \{x \mid x \in (0, L)\}$.
A bar of length $L = 10$ and elastic constant $E = 1$ is considered. The nodal discretization is shown in Figures 1 and 2. In Figure 1, nodes 1-4 and 26-29 are FE nodes while nodes 5-25 are EFG nodes (support size $[d_{mI}]$ for nodes 5 and 25 is set to 1.02). It is seen that the stresses are markedly different from the exact solution and hence the numerical solution in Figure 1 fails to satisfy the patch test.

In Figure 2, nodes 1-5 and 25-29 are FE nodes while nodes 6-24 are EFG nodes. The axial stress is plotted for different values of d_{ratio} and quadrature (Q). The result for $Q5$ with $d_{ratio} = 3$ matches the exact solution to at least six significant digits. The norm-values for the different cases is presented in Table 1. The results shown in Figure 2 are a consequence of accurate integration being carried out in the evaluation of the stiffness matrix: the fact that nodal domains of influence terminate at cell boundaries facilitates the integration and hence the satisfaction of the patch test.

Table 1: $\mathcal{L}_2$- and $\mathcal{H}^1$-norms

Nodes 5&25	d_{ratio}	**Quadrature**	$\mathcal{L}_2$	$\mathcal{H}^1$
EFG	3.0	5	6.08×10^{-3}	4.14×10^{-1}
FE	3.0	4	8.15×10^{-7}	9.18×10^{-5}
FE	3.0	5	1.70×10^{-8}	2.53×10^{-6}
FE	3.0	6;2;4	1.06×10^{-6}	1.24×10^{-4}
FE	2.0	5	2.60×10^{-7}	3.54×10^{-5}

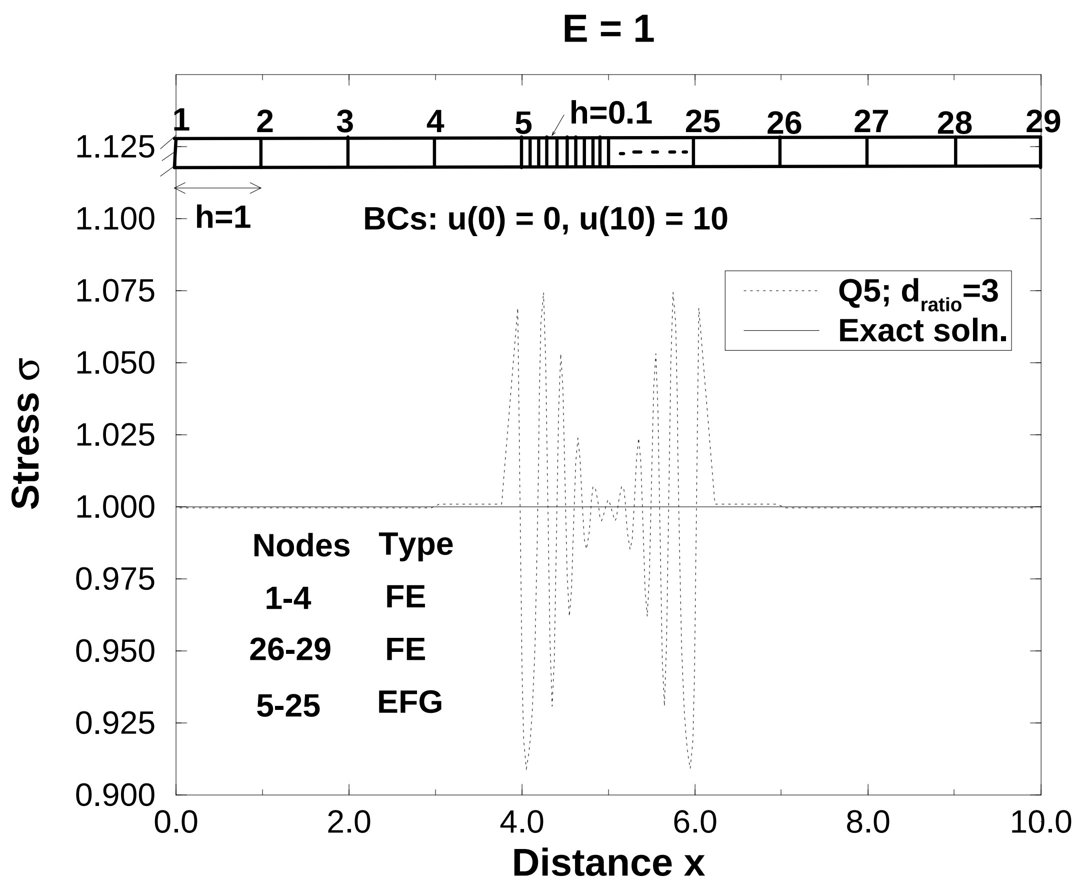

Figure 1: Comparison of numerical and exact solution — σ_x versus x

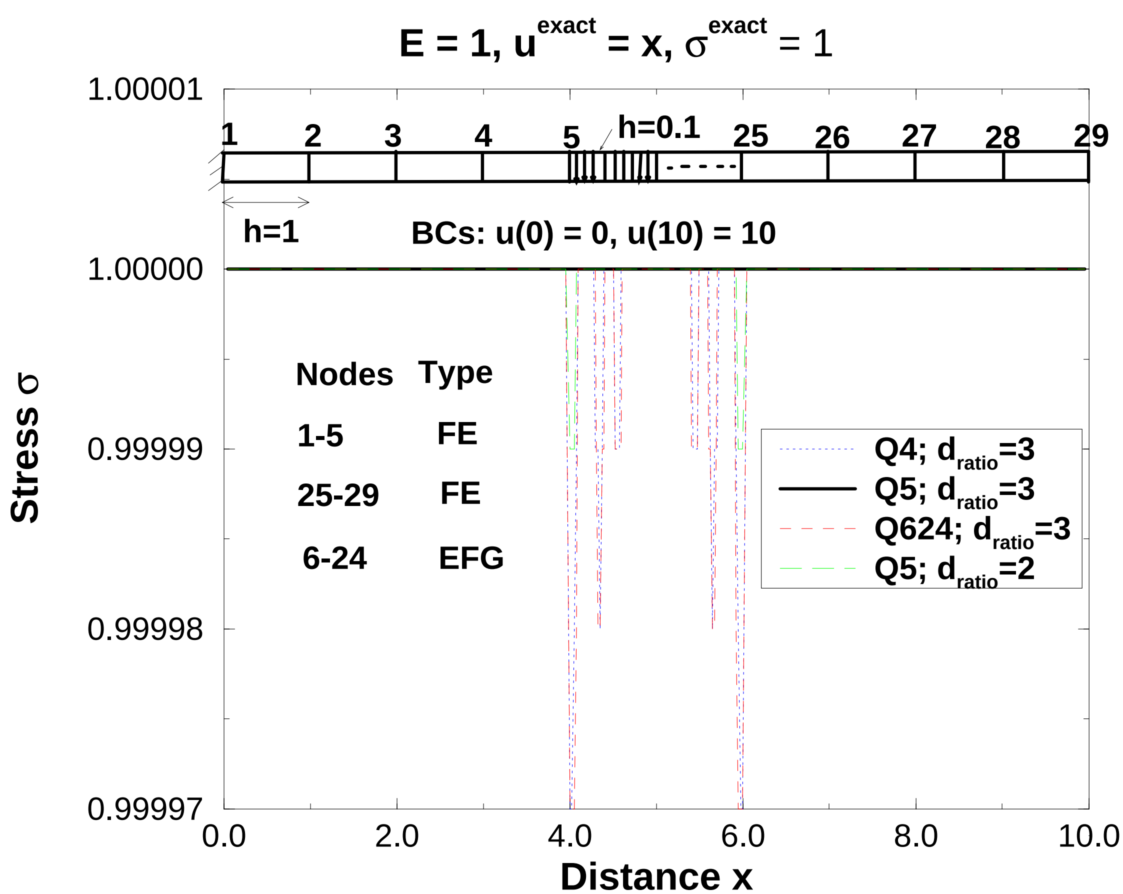

Figure 2: Numerical solution: σ_x versus x