The Natural Element Method in Solid Mechanics

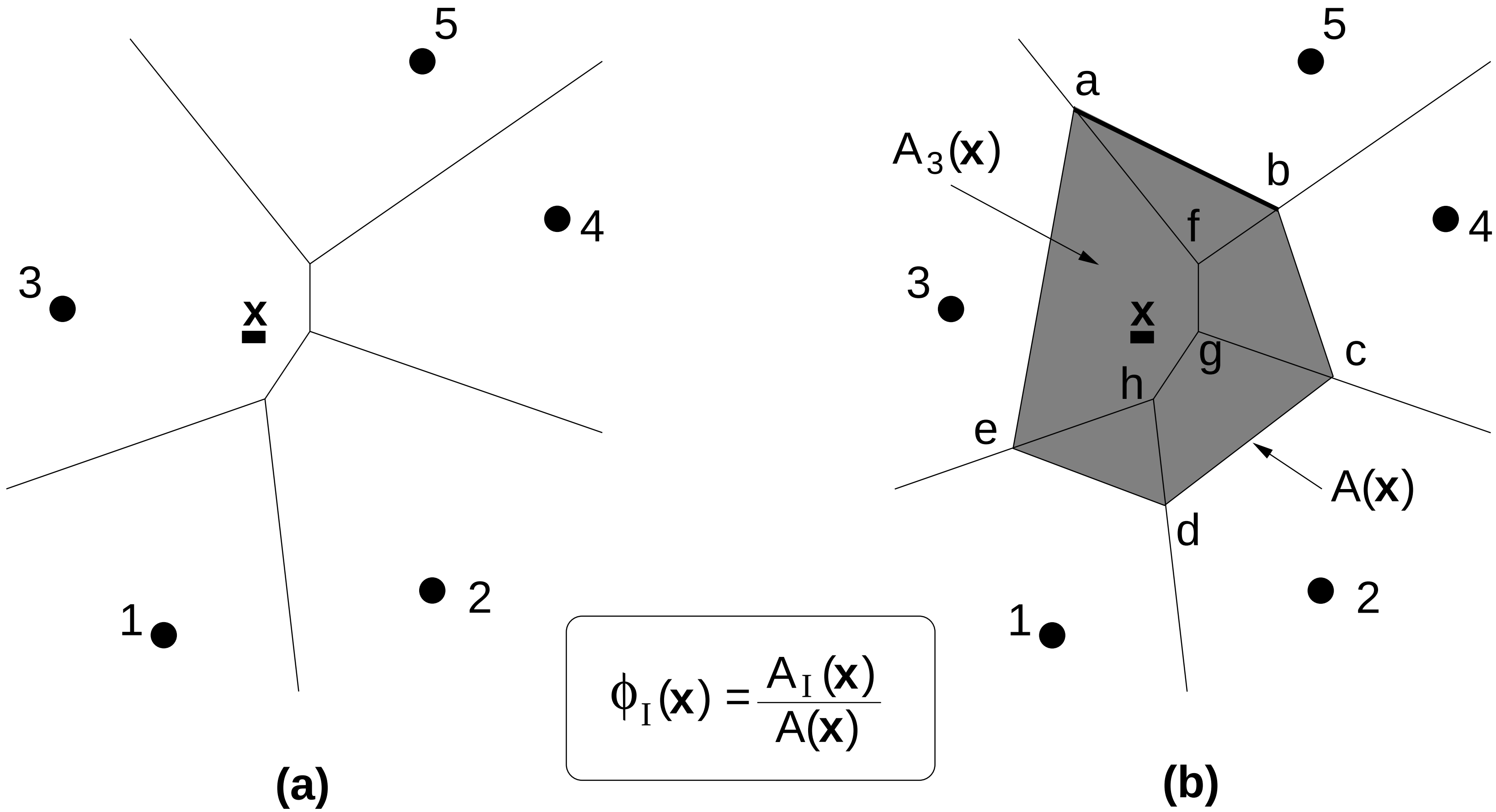
N. Sukumar

Theoretical and Applied Mechanics
Northwestern University

Ph.D. Defense
May 8, 1998

Outline

- Galerkin Methods
- Natural Neighbor Interpolation
- Natural Element Method (NEM)
- C^1 Natural Neighbor Interpolant
- Mixed Formulation for NEM in Linear Elasticity
- Conclusions



$$\phi_I(\mathbf{x}) = \frac{A_I(\mathbf{x})}{A(\mathbf{x})}$$

(a) Original Voronoi Diagram

(b) New Voronoi cell about x

Natural Element Method (NEM)

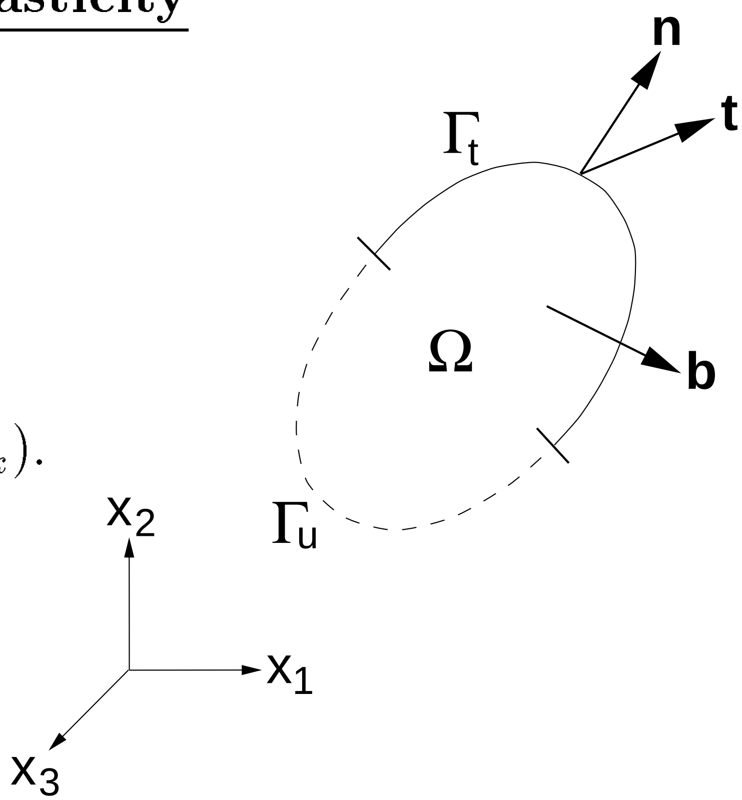
Governing Equations of 2D Elasticity

$$\sigma_{ij,j} + b_i = 0 \quad \text{in} \quad \Omega,$$

$$\sigma_{ij} = C_{ijkl}(\varepsilon_{kl} - \varepsilon^*_{kl}),$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),$$

$$C_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}).$$



Boundary Conditions

$$u_i = \bar{u}_i \quad \text{on} \quad \Gamma_u\,,$$

$$t_i = \sigma_{ji}n_j = \bar{t}_i \quad \text{on} \quad \Gamma_t.$$

Conclusions

- Natural neighbor coordinates, which yield a linearly complete approximation, are based on an appealing geometric construct for a given set of nodes in the plane.
- Applications of the natural element method in 2D elastostatics were met with considerable success.
- C^1 natural neighbor interpolant was proposed for higher-order PDEs, such as the biharmonic equation.
- A mixed natural element method formulation for compressible as well as incompressible elasticity was presented.
- Future work on NEM with focus on nonlinear mechanics and 3D would further the understanding and potential of the method.