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The Natural Element Method in Solid Mechanics

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ABSTRACT

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The Natural Element Method (NEM) is a recently proposed novel numerical tool for the solution of partial differential equations. In this work, the development and application of the natural element method to two-dimensional elliptic boundary value problems in solid mechanics is presented.

We assume the discrete model of a body $\Omega \subset \mathbb{R}^2$ consists of a set of distinct nodes N, and a polygonal description of the boundary $\partial\Omega$. In the natural element method, the trial and test functions are constructed using natural neighbor interpolants. These interpolants are based on the Voronoi tessellation of the set of nodes N. The NEM interpolant is strictly linear between adjacent nodes on the boundary of the convex hull, which facilitates imposition of essential boundary conditions. A methodology to model material discontinuities and non-convex bodies such as cracks is described. A standard displacement-based Galerkin procedure is used to obtain the discrete system of linear equations. Application of NEM to various problems in two-dimensional elastostatics is presented.

The construction and computational implementation of a C^1 natural neighbor interpolant for fourth-order elliptic PDEs is presented. By embedding natural neighbor interpolants in the surface representation of a Bernstein-Bézier cubic simplex, a C^1 interpolant is realized (Farin, 1990b). We present the C^1 formulation and pro-

pose a computational methodology for its numerical implementation for the solution of PDEs. Numerical results for the biharmonic equation with Dirichlet boundary conditions are presented.

A mixed formulation for the natural element method in linear elastostatics is presented. A displacement-pressure mixed formulation is adopted with displacements interpolated by C^0 natural neighbor interpolants; C^0 and C^{-1} interpolation schemes are considered for the interpolation of the pressure. The mixed C^0–C^{-1} NEM formulation alleviates locking in the near incompressible limit ($\nu \to 0.5$) for the elastostatic boundary value problem; moreover, convergence rates in displacement and energy are optimal for all $\nu \in [0, 0.5)$. Results for benchmark problems in compressible and incompressible elasticity are presented.

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