Enriched Natural Neighbor Interpolants for Two-Dimensional Crack Modeling

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Abstract

A numerical technique to model arbitrary cracks in two-dimensions within the framework of meshfree methods is proposed. The methodology used closely follows that adopted in the eXtended Finite Element Method (X-FEM) [1, 3] for modeling arbitrary discontinuities within the finite element framework. The extension of the technique to meshfree methods is presented, and in this particular instance, natural neighbor-based interpolants [2, 4] are used to construct the trial and test functions in the displacement-based Galerkin implementation. Natural neighbor (Sibson and non-Sibsonian) interpolants are based on the Voronoi diagram, a fundamental geometric construct that partitions a given set of nodes in $\mathbb{R}^d$. Imposition of essential boundary conditions in natural neighbor Galerkin methods can be carried out as in finite elements, unlike most other meshfree methods, where satisfaction of essential boundary conditions for a linear field can not be met exactly. In addition, for an irregular arrangement of nodes in 2-d or 3-d, natural neighbor-based interpolants provide an appealing means to construct robust approximations. In the present work, as in the X-FEM, a discontinuous function and the two-dimensional asymptotic crack-tip displacement fields are added to the natural neighbor approximation to account for the crack using the notion of partition of unity. This enables the domain to be modeled independent of the crack topology. Since the support of the nodal shape functions as well as many other properties of natural neighbor interpolants bear much similarity to finite elements, an efficient algorithm emerges for the implementation of the method. Benchmark crack problems in two-dimensions are solved and compared to analytical solutions to demonstrate the accuracy of the technique.

References

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